Design of oscillation-based test structures for active RC filters

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Abstract: The oscillation test methodology is presented for active RC filters. The authors develop general guidelines for the design of the oscillation-based test structures and describe in more detail test transformations of filter configurations which are most frequently used in practice. Frequency measurements performed on some implemented test structures and fault detection experimental results are included. The oscillation-based test described in the paper is primarily intended for field testing. In some cases, it can also be used as a functional go no-go test in production. The presented test structures can also be employed for stimulus generation in a built-in self-test.

1 Introduction

Oscillation-based test strategy [1], proposed some years ago, has been applied to testing integrated operational amplifiers [2], digitally programmable switched-current biquadratic filters [3], and analogue-to-digital converters [4]. In oscillation-based testing, the circuit that we want to test is transformed into an oscillating circuit and the frequency of oscillation is measured. The frequency of a fault-free circuit is taken as a reference value. Discrepancy between the oscillation frequency of a circuit-under-test and the reference value indicates possible faults. The way how to put a circuit into oscillation and the choice of the frequency of oscillation depend on the employed fault-detection procedure. Fault-detection can be performed as a built-in self-test or in the frame of an external tester. In the first case, the original circuit is modified by inserting some test control logic which provides for the oscillation during the test mode. In the second case, the oscillation is achieved by an external feedback loop network which is normally implemented as a part of a dedicated tester.

In this paper we derive structures for oscillation-based test of active RC filters. For this class of circuits, different DFT methodologies have been proposed [5, 6] after the initial work of [7]. In our approach, the filter stage is put into oscillation at the frequency of the pole (and at the frequency of the zero, if its value differs from zero or infinity). The filter stage is accepted as fault-free if the measured frequency lies close to the nominal value (i.e. within the acceptance range corresponding to the required specifications). This paper is an extension of our work [8, 9].

The oscillation-based test described in this paper can be regarded as complementary to the existing methodologies and is primarily intended for field testing where a field engineer can configure the circuit in a test mode and measure the output frequency with a counter or oscilloscope. In some cases, it can also be used as a functional go no-go test in production.

2 Preliminaries

Consider the general form of the second-order transfer function, where  \( \omega_z \) and  \( \omega_p \) are the undamped frequencies of the zero and the pole,  \( H \) is a constant, and  \( Q_z \) and  \( Q_p \) are the qualities of the zero and the pole:

\[
\frac{V_2(s)}{V_1(s)} = \frac{s^2 + \frac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}
\]

The relation between the pole located at  \( p_{1,2} = \sigma \pm j\omega \) and the quantities  \( \omega_z, \omega_p, Q_z, Q_p \) is given by

\[
p_{1,2} = \sigma \pm j\omega = -\frac{\omega_p}{2Q_p} \pm j\frac{\omega_p}{2Q_p} \sqrt{4Q_p^2 - 1}
\]

To put the network into oscillation with constant amplitude the pole must be placed on the  \( j\omega \)-axis. Consequently, if  \( \omega_p^2Q_p = 0 \), the network will oscillate with the resonant frequency  \( \omega_r \). This condition is satisfied when the quality is sufficiently high (ideally infinite). Our goal is to put the filter stage into oscillation at the frequency of the undamped pole which inherently reflects the overall characteristics of the stage. Both  \( \omega_z \) and  \( Q_z \) are functions of the elements of the network. To keep the frequency of the oscillation at the undamped pole, the quality  \( Q_p \) should be increased only by changing the values of the components which do not affect  \( \omega_z \).

Taking this into account we derive the oscillation-based test transformations of some filter configurations that are frequently used in practice. Presented solutions have been obtained under the assumption that the employed op-amps are ideal. In reality, the gain and bandwidth of op-amps are limited. This in principle does not change the general approach but must be considered in practical realisations.

3 Test transformations of active RC filters

3.1 Multiple amplifier filters

3.1.1 Resonator active filter (biquad filter): The configuration of the resonator active filter which can be used for realisation of the lowpass and bandpass character-
istics is shown in Fig. 1. The bandpass transfer function (considered in our case) is given by

$$\frac{V_2(s)}{V_1(s)} = \frac{-\frac{R_0}{R_4C_1R_3s}}{s^2 + \frac{1}{R_1R_3C_1} + \frac{1}{R_2R_3C_1C_2}} \quad (3)$$

The frequency of the pole and quality factor are given by the expressions

$$\omega_p = \sqrt{\frac{R_0}{R_2R_3R_1C_1C_2}} \quad (4)$$

$$\frac{1}{Q_p} = \frac{1}{R_1} \sqrt{\frac{R_2R_3C_2}{C_1}} \quad (5)$$

From the above expressions, we can see that the condition for oscillation $V_2(s) / V_1(s) \rightarrow 0$ without affecting $\omega_p$ will be satisfied if $R_1 \rightarrow \infty$. As a result, switch $S_1$ is introduced to disconnect $R_1$ during the test mode, as shown in Fig. 1.

**Fig. 1** Resonator active filter (with switch $S_1$ for test mode)

**Fig. 2** KHN state-variable filter (with switch $S_1$ or $S_2$ for test mode)

### 3.1.2 KHN state-variable filter

The configuration of the KHN state-variable filter, another representative of the group of multiple amplifier filters, is shown in Fig. 2. It can be used for realisation of lowpass, bandpass and highpass characteristics. The bandpass transfer function (considered in our case) is given by

$$\frac{V_2(s)}{V_1(s)} = \frac{1 + \frac{R_0}{R_4s}}{1 + \frac{R_0}{R_4s} + s^2 + \frac{1}{R_1R_3C_1} + \frac{1}{R_2R_3C_1C_2}} \quad (6)$$

and the frequency of the pole and quality factor are given by

$$\omega_p = \sqrt{\frac{R_0}{R_1R_3R_2C_1C_2}} \quad (7)$$

$$\frac{1}{Q_p} = \frac{1}{R_1} \sqrt{\frac{R_2R_3C_2}{R_0R_1C_1}} \quad (8)$$

As in the previous case, the oscillation condition will be satisfied if $R_0 / R_4 = 0$. The oscillation test mode can be achieved either by opening $R_4$ or shorting $R_6$. For illustration, the first possibility is shown in Fig. 2, where $S_1$ is used to disconnect $R_4$.

### 3.2 Single amplifier filters

The general configuration given in Fig. 3 has the voltage transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{K N_{31}(s)}{N_{33}(s) - K N_{32}(s)} \quad (9)$$

In the above expression $N_{31}(s) = Y_1Y_3$, $N_{31}(s) = Y_1Y_2$, $N_{31}(s) = Y_1Y_3$ and $N_{32}(s) = Y_3Y_4$, $N_{32}(s) = Y_3Y_4$. $K$ is the gain of the amplifier. The denominator of the voltage transfer function can be rearranged to the form $(s + \sigma_1)(s + \sigma_2) + K$, where $\sigma_1$, $\sigma_2$ and $K$ are functions of the passive network elements. The poles of the voltage function will move into the right half of the s-plane including the j$\omega$-axis when $K \geq (\sigma_1 + \sigma_2)$. Hence, by choosing an appropriate value of $K$ we can put the network into oscillation.

**Fig. 3** General configuration of single amplifier filters

3.2.1 Single amplifier positive gain filter: The Sallen and Key filter, a representative of single amplifier positive gain filters, is shown in Fig. 4. Its transfer function is given by

$$\frac{V_2(s)}{V_1(s)} = \frac{K}{s^2 + s \left(\frac{1}{R_1R_3C_1} \right) + \frac{1}{R_1R_2C_2C_4}} \quad (10)$$

and the quality factor

$$\frac{1}{Q_p} = \frac{\sqrt{R_3C_4}}{R_1R_2C_2} + \frac{\sqrt{R_1C_4}}{R_3C_2} + (1 - K) \frac{R_1C_3}{R_3C_4} \quad (11)$$

By putting $1/Q_p = 0$ we get $K = (R_3C_4 + R_1C_4 + R_1C_2)/R_2C_2$.

The amplifier with gain $K$ can be realised in different ways. One of the standard realisations is shown in Fig. 5, where $K = 1 + R_6/R_4$. Some external control of the value of $R_6/R_4$ must be provided to obtain the required value of $K$ in the test mode. The notch filter, another representative of this group, is shown in Fig. 6, and its transfer function is
given by
\[
\frac{V_2(s)}{V_1(s)} = K \left[ s^2 + \frac{C_3 + C_0}{R_1 R_2 C_2 C_3} + \frac{C_2 + C_0}{R_2 R_3 C_2 C_3} + \frac{C_3 + C_1}{R_1 R_3 C_2 C_3} \right]
\]
(12)

The frequencies of the pole and zero are
\[
\omega_z = \omega_p = \omega_0 = \sqrt{\frac{C_3 + C_0}{R_1 R_2 C_1 C_2 C_3}}
\]
(13)

From the above expression it is apparent that the same elements influence the pole and zero positions.

![Fig.5](image)

**Fig.5** Realisation of the lowpass Sallen and Key filter

\[ K = 1 + R_p R_1 \]

![Fig.6](image)

**Fig.6** Notch filter

Similar to the Sallen and Key filter, we can put the circuit into oscillation by choosing the appropriate value of \( K \). Note, however, that even when the passive elements are in perfect adjustment, the real amplifier bandwidth causes dissimilar effects on the pole and zero positions. In contrast to the pole position, the zero position proves to be independent of the gain of the amplifier [10]. For example, simulations of the filter stage at \( f_0 = 1.6 \text{kHz} \) indicate that for the opamp μA741 the frequency of the pole is shifted for \( -0.6\% \), while for LS204 the shift is \( -0.1\% \).

### 3.3 Single amplifier infinite gain filters

For the group of single amplifier infinite gain filters we assume the value of \( K \) approaching infinity. The voltage transfer function of the general configuration becomes \(-N(s)/A_1(s)\). As an example of this group we take a lowpass filter shown in Fig. 7 with the voltage transfer function
\[
\frac{V_2(s)}{V_1(s)} = \frac{R_2}{s^2 + \frac{1}{C_0 \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)} + \frac{1}{R_2 R_3 C_0 C_1}}
\]
(14)

The quality factor and the frequency of the pole are given by
\[
\frac{1}{Q_p} = \sqrt{\frac{C_0}{C_3}} \left( R_1 + \sqrt{\frac{R_2 R_3}{R_1}} \right)
\]
(15)

From the expression presenting the quality factor we can see that the oscillation condition cannot be met by modifying values of the circuit elements without affecting \( \omega_0 \). A feedback loop network is required to shift the phase to \( -360^\circ \) at the frequency of the pole. In practice, a noninverting integrator can be used to put the circuit into oscillation. In a similar way we could derive the solutions for bandpass and highpass filter stages. It can be shown, for example, that the required feedback loop network of a bandpass filter stage is realised by an inverter.

![Fig.7](image)

**Fig.7** Lowpass single amplifier filter

### 3.4 Active RC filters with active two-port elements

This group of filters comprises classes of active RC filter configurations which use active two-port network elements [11] like, for example, generalised immittance converter (GIC), negative immittance converter (NIC), positive immittance converter (PIC), etc. As in previous cases, the derivation of oscillation-based test structures requires the analysis of the dependency of the quality and the resonant frequency of the filter poles on the changes of the filter elements. The generalised approach is beyond the scope of this paper. Just for illustration we briefly refer to the active RC filter with frequency dependent negative resistor (FDNR) which is widely used in the high-performance active filter realisations. The transfer function of a lowpass filter with FDNR is given by
\[
\frac{V_2(s)}{V_1(s)} = \frac{R_2}{s^2 + \frac{R_2}{C_1 C_2 R_3 R_5} + \frac{R_1}{C_1 C_2 R_3 R_5 (R_1 + R_2)}}
\]
(17)

From the denominator we observe that the position of the poles is on the \( jo \) axis; hence the circuit is potonially unstable. In practice the instability may not occur due to the fact that the above expression has been derived for the ideal case, while in reality the finite values of opamp gains will cause the poles to shift slightly into the left half of the complex plane. Since the coefficient of the term \( s \) in the denominator is missing there is no component available for increasing the quality of the pole. Consequently, an additional feedback network providing for oscillation with the frequency of the pole,
\[
\omega_p = \sqrt{\frac{R_4}{C_1 C_2 R_3 R_5 (R_1 + R_2)}}
\]
(18)

is needed, as shown in Fig. 8. One of the possible solutions is to use an allpass equaliser as the additional feedback network [12]. The configuration enables measurements of the frequency of the pole (jumper in 2-4 position) as well as the frequency of the zero (jumper in 3-4 position).
4.1 Bandpass resonator filter

Two different bandpass resonator circuits making part of the signal detector in a telecommunication application were taken for the first case study. In the analysis of the operation in the oscillation test mode, each of them was implemented with two different opamps: LS204 and LM2902. Their operation was first simulated and the computed values of frequencies are given in Table 1. In the Table, the first circuit is denoted by filter 1 and the second by filter 2. The computed values represent the results for the case of ideal opamps (computed from the expression for \( f_0 \)) and for LS 204 and LM 2902 modelled by the dominator-pole models.

<table>
<thead>
<tr>
<th>Table 1: Computed oscillation frequencies of two-filter circuits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Ideal opamp</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Dominant pole model</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Next, circuit characteristics were measured in the normal mode and in the test mode. Table 2 gives the measured values of phase and gain at the frequency of the pole (in the normal mode) and the frequency of oscillation (in the test mode). The first measurement was performed for the circuit operating in the normal mode (without \( S_1 \)). The test mode was achieved by opening \( R_1 \) (laser cut of the resistor layer) and the frequency of oscillation was measured. In the next step, external resistor \( R_3 \) and test switch \( S_1 \) (HEF4066D) were connected as shown in Fig. 1. Measurements in the normal and the test mode were performed. The results indicate that the effect of the introduced MOS test switch in this case on the circuit characteristic in the normal mode can be neglected. Furthermore, the frequency of oscillation in the test mode with a built-in MOS switch is very close to the frequency of the pole in the normal mode, which provides the basis for the implementation of a consistent oscillation-based test in practice.

In the following we give some experimental evidence regarding the efficiency of the oscillation-based test in fault detection. Different faults were inserted into the two filter circuits and the oscillation frequency in the test mode was measured. Inserted faults were chosen such that they reflected fault situations that may occur in the production process. In our case, the thick film resistors are laser trimmed and the circuit is actively adjusted. Possible open connections, shorts or missing components are detected during the adjustment. What remains are parametric faults which may in some cases result in a component value considerably different from the nominal. Finally, the encapsulation process may introduce some hard faults (that manifest similar to a short circuit).

Fault detection results for our experimental circuits are given in Table 3 (for filter 1 implemented with LS204) and Table 4 (for filter 2 implemented with LM2902). Faults were inserted by replacing components or placing additional components in parallel to the existing ones. Faults in capacitors correspond to a hard fault (capacitor shorted) or to a wrong component. Faults in resistors reflect failures in the laser trimming process and encapsulation. Most of the faults could be detected from the measured frequency of the oscillation. The exception was faulty \( R_1 \), which is dis-
connected in the test mode. In this case, the fault was detected by measuring the gain at \( f_{osc} \) in the normal mode. For LS204, the gain was 19.2 dB instead of the expected 21.5 dB. For LM2902, the gain was -0.89 dB instead of 22.7 dB.

Table 3: Fault detection results for filter with LS204

<table>
<thead>
<tr>
<th>Component</th>
<th>Faulty value</th>
<th>Correct value</th>
<th>( f_{osc} ) Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1</td>
<td>400 kΩ</td>
<td>598.18 kΩ</td>
<td>1227</td>
</tr>
<tr>
<td>R_2</td>
<td>427 kΩ</td>
<td>11.36 kΩ</td>
<td>6665</td>
</tr>
<tr>
<td>R_3</td>
<td>128 kΩ</td>
<td>11.36 kΩ</td>
<td>4002</td>
</tr>
<tr>
<td>R_4</td>
<td>9.06 kΩ</td>
<td>11.36 kΩ</td>
<td>1664</td>
</tr>
<tr>
<td>C_1</td>
<td>20.8 nF</td>
<td>10 nF</td>
<td>921</td>
</tr>
<tr>
<td>C_2</td>
<td>shorted</td>
<td>no fault</td>
<td>no oscillation</td>
</tr>
</tbody>
</table>

Table 4: Fault detection results for filter with LM2902

<table>
<thead>
<tr>
<th>Component</th>
<th>Faulty value</th>
<th>Correct value</th>
<th>( f_{osc} ) Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1</td>
<td>39.22 kΩ</td>
<td>578.06 kΩ</td>
<td>1202</td>
</tr>
<tr>
<td>R_2</td>
<td>110 kΩ</td>
<td>11.00 kΩ</td>
<td>424</td>
</tr>
<tr>
<td>R_3</td>
<td>4.8 kΩ</td>
<td>10 kΩ</td>
<td>303</td>
</tr>
<tr>
<td>R_4</td>
<td>7.48 kΩ</td>
<td>10 kΩ</td>
<td>1475</td>
</tr>
<tr>
<td>C_1</td>
<td>shorted</td>
<td>10 nF</td>
<td>468271</td>
</tr>
<tr>
<td>C_2</td>
<td>20 nF</td>
<td>10 nF</td>
<td>924</td>
</tr>
</tbody>
</table>

Table 5: Fault detection results for Sallen and Key filter in Fig. 11

<table>
<thead>
<tr>
<th>Inserted fault</th>
<th>( f_{osc} ) Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1 = 14.31 kΩ</td>
<td>no oscillation</td>
</tr>
<tr>
<td>R_2 = 15.74 kΩ</td>
<td>1045.3 s (0.6%)</td>
</tr>
<tr>
<td>R_3 = 15.06 kΩ</td>
<td>1032 s (-0.6%)</td>
</tr>
<tr>
<td>R_4 = 1 MΩ</td>
<td>60.3</td>
</tr>
<tr>
<td>R_4 disconnected</td>
<td>no oscillation</td>
</tr>
<tr>
<td>1 MΩ connected parallel to C_1</td>
<td>no oscillation</td>
</tr>
<tr>
<td>1 MΩ connected parallel to C_2</td>
<td>no oscillation</td>
</tr>
<tr>
<td>C_1 shorted</td>
<td>no oscillation</td>
</tr>
<tr>
<td>C_2 shorted</td>
<td>no oscillation</td>
</tr>
<tr>
<td>R_2 shorted</td>
<td>1835</td>
</tr>
</tbody>
</table>

4.2 Sallen and Key filter

The next example is a high-pass Sallen and Key filter shown in Fig. 11 with the following values of its components: \( R_4 = 15.9 kΩ \), \( C_1 = 9.64 nF \) and \( C_2 = 9.65 nF \). The employed opamp was LS204. In the normal operation, \( R_4 = 21 kΩ \) and \( R_2 = 12.5 MΩ \). In the test mode, by changing

4.3 Lowpass FDNR filter

The last example is taken from the production of a high-precision low-pass filter with FDNR (equal-ripple, passband max. 0.25 dB, min. attenuation 60 dB, the slope of selectivity \( k_s \), 10.3) realised by a cascade of stages shown in Fig. 8. The circuit was produced in thick-film technology with LS204 opamps. The additional feedback loop network was realised with a CA3110 opamp.

During active adjustment by laser trimming of resistors most of the faults from the previous steps in the production process (printing of the metal and resistor layers, laser trimming of resistors, component assembly, etc.) are detected. The final test aims at detecting faults in resistors that are actively adjusted and faults that may occur in the packaging process. An oscillation-based go no-go test is an effective means for detecting these faults. The test includes measurements of the frequency of the pole and the frequency of the zero. Table 6 gives some results to illustrate the performed test process. The oscillation-based test comprised measurements of the frequency of the pole \( f_p \) and the
frequency of the zero \( f_z \). For each circuit-under-test, the
relative difference from the reference frequency value was
computed. In accordance with the given requirements,
acceptance/reject margins were taken at \( \Delta f_p = 0.2 \% \) for
the frequency of the pole and \( \Delta f_z = 1 \% \) for the frequency
of the zero.

The first three circuits are taken from the list of fault-free
circuits. Since the majority of faults are detected at the
earlier steps, the number of the remaining faults is relatively
small. Circuits 4-6 are representatives of the faults in resistance
related to the laser trimming process. Circuit 4 corre-
sponds to a faulty \( R_4 \) (2.7\% lower than the nominal value)
and circuit 6 to a faulty \( R_6 \) (7.5\% lower than the nominal
value). In sample 5, \( R_4 \) is faulty (8.2\% lower than the
nominal value). From eqn. 17 it is obvious that this fault does
not affect the frequency of the zero.

5 Conclusion

We have presented the oscillation-based test transforma-
tions of active RC filter configurations. Attention has been
directed to the conditions for practical realisations of test
structures oscillating at the frequency of the undamped pole of
the filter stage which inherently reflects the overall character-
istics of the stage. By measuring this frequency in the test
mode, a simple and effective functional test can be performed.
Of course, due to the circuit modifications, faults in some components that affect circuit normal operation
(for instance, the resistor \( R_4 \) in the biquad circuit, Section
4.1) can be missed. Other means for detecting these faults
must be provided. On the other hand, when an external
feedback network is provided to put the filter stage into
oscillation (for example, test structure of \( \text{FDNR} \) filter
described in Section 3.4) the above problem does not exist.

Proposed test structures can also be employed for stimu-
lus generation in a built-in self-test. For example, by vary-
ing the value of \( K \) in the lowpass Sallen and Key filter in
Fig. 5, frequencies smaller than the frequency of the
undamped pole can be generated.

Proposed transformations have been derived under the
assumption that the employed opamps are ideal. In reality,